Acquiring Competence from Performance Data
Online learnability of OT and HG with simulated annealing

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The language acquisition problem
Learning from competence?
Learning from performance!
Distance of teacher’s and learner’s performance
Overview

1. Modelling linguistic performance
2. Learning
3. Results
4. Conclusions
Overview

1. Modelling linguistic performance

2. Learning

3. Results

4. Conclusions
Errors and mental computations

A grammar is a Harmony function on the candidate set, defined by the ranked constraints.
Global optimum: more harmonic than all other candidates.
Local optimum: more harmonic than its neighbours.

Optimality Theory

grammar competence model
grammatical form = $\mathcal{F}$ (globally) optimal candidate

SA-OT implementation performance model
produced forms = globally or locally optimal candidates
**Competence and performance models**

\[ SF(U) = \arg \text{opt} \ H(w) \]

(w ∈ Gen(U))

**Competence models:**
- \( C_i(w) \) elementary functions on the candidates (“constraints” – a misnomer).
- Optimality Theory: \( H(w) = (C_n(w), ..., C_1(w)) \)
  \( \arg \text{opt}: \text{lexicographic order.} \)
- \( q \)-Harmony Grammar: \( H(w) = C_n(w) \cdot q^n + ... + C_i(w) \cdot q \).
  
  Large \( q \): OT-like strict domination.
  
  Small \( q \): ganging-up effects.

**Performance models:**
- Exhaustive search: returns global optimum.
- Simulated annealing: returns some local optimum.
  - Run slowly: frequently the globally optimal one.
  - Run quickly: global opt. less frequent, more often performance errors.
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Online learning algorithms

Constraint $C_i$ has rank $r_i$.

In each learning cycle: learning data (winner) produced by teacher compared to form produced by learner (loser).

**Update rule:** update the rank $r_i$ of every constraint $C_i$, depending on whether $C_i$ prefers the winner or the loser.

- Boersma (1997): increase rank by $\epsilon$ if winner-preferring; decrease rank by $\epsilon$ if loser-preferring constraint.

- Magri (2009): increase rank of all winner-preferring constraints by $\epsilon$; decrease rank of highest ranked loser-preferring constraint by $W \cdot \epsilon$, where $W$ is the number of winner-preferring constraints.
Learn until performance converges

- Convergence of performance, and not of competence. Child may acquire different grammar.
- Sample of teacher vs. sample of learner (sample size = 100).
- **Convergence criterion:** $JSD$ between sample produced by target grammar and sample produced by learner’s current grammar $\leq$ average $JSD$ of two samples produced by target grammar.

**Jensen-Shannon divergence:** measures the “distance” of two distributions

$$JSD(P\parallel Q) = \frac{D(P\parallel M) + D(Q\parallel M)}{2}$$

where $D(P\parallel Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$ (relative entropy, Kullback-Leibler divergence), $M(x) = \frac{P(x) + Q(x)}{2}$.

- Symmetric: $JSD(P\parallel Q) = JSD(Q\parallel P)$. Non-negative: $JSD(P\parallel Q) \geq 0$. $JSD(P\parallel Q) \leq 1$.
- $JSD(P\parallel Q) = 0$ if and only if $P(x) = Q(x)$, $\forall x$. $JSD(P\parallel Q) = 1$ if and only if $P(x) \cdot Q(x) = 0$, $\forall x$.
- Same language: $JSD(L_t\parallel L_l) = 0$. Not a single overlap: $JSD(L_t\parallel L_l) = 1$. 
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Results: number of learning steps until convergence

- 2000 times learning (rnd target, rnd underlying form) per grammar type × production method × learning method.
- Measure the number of learning steps until convergence.
- Distribution of the number of required learning steps:

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(1st quartile ; median ; 3rd quartile)
Methodological notes

Paradigm:

- Measure number of learning steps until converging performance.
- Statistics on the distribution of the required learning step number.
- Under different learning conditions.
- Distributions have extremely long tails.

Significance of differences: using non-parametric tests.

Does learning speed depend on initial grammar? On learning data?

Run two learners learning the same target grammar:

- with same initial grammar: strong correlation in nr. of learning steps.
  Learning data not the same: slightly decreased correlation.
- with different initial grammars: correlation (almost) lost.

Long tail: children must start with same initial grammar, but need not receive same (correct or erroneous) data (if learning algorithm is correct).
Conclusions

Proposed paradigm for the learnability of a grammar framework:

- Competence = grammar framework (e.g., OT or HG).
- Performance = imperfect implementation of competence model.
- Learning from performance data, only partially reflecting competence.
- Learner does not have access to teacher’s competence directly: converge on performance.
- Convergence measure using Jensen-Shannon divergence.
- Argument for same initial grammar in children?

Implemented on OTKit.
Thank you for your attention!

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Tools for Optimality Theory
http://www.birot.hu/OTKit/

Work supported by: