From Harmonic Grammar to Optimality Theory:
Production and maturation in *q*-HG

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Optimality Theory (OT) and *Harmonic Grammars* (HG) are sister theories. While the connectionist version of HG predates OT, symbolic HG has only moved to the foreground of scholarly interest in the last decade. This abstract introduces a variant of HG, called *q*-HG, in order to interpolate between the two theories. First, the role of the parameter *q* is discussed in competence and performance. Subsequently, child language *maturation* will be proposed to be changes in the value of *q*.

Both OT and HG introduce elementary functions $C_k(x)$ (where $k = 1 \dots n$), called *constraints* for historical reasons, on the set $\mathfrak{X}$ of candidates. This set $\mathfrak{X}$ is the image of the set of underlying forms $\mathfrak{U}$, postulated to be universal by the *Richness of the Base Principle,* under the one-to-many mapping GEN, also postulated to be universal. Although there are some exceptions to it in the linguistic literature, we shall suppose in this abstract that the range of the constraints are the non-negative integers ("numbers of stars").

Both theories optimize an objective function $H(x)$ called *Harmony*. In OT, this function is a vector – known as a row in an OT tableau – built from the constraint violations $C_k(x)$, and optimization is with respect to lexicographic order. In HG, this function is a real number, the weighted sum of the constraint violations, optimized with respect to the arithmetic *greater than* relation. In *q*-HG, the weights are exponents of some base $q > 1$:

$$H_{\mathrm{OT}}(x) = \left(-C_n(x), -C_{n-1}(x), \dots, -C_k(x), \dots, -C_1(x)\right)$$
$$H_{\mathrm{HG}}(x) = -\sum_{k=1}^{n} w_k \cdot C_k(x)$$
$$H_q(x) = -\sum_{k=1}^{n} q^k \cdot C_k(x)$$

In all theories, the grammatical output (or surface form) corresponding to input (or underlying form) $u \in \mathfrak{U}$ is defined as the candidate(s) optimizing the objective function:

$$\mathrm{SF}_{\mathrm{OT}}(u) = \arg \max_{x \in \mathrm{GEN}(u)} H_{\mathrm{OT}}(x)$$
$$\mathrm{SF}_q(u) = \arg \max_{x \in \mathrm{GEN}(u)} H_q(x)$$

What is the role of q in *q*-HG? How does *q*-HG contribute to our understanding of the relation between HG and OT? This abstract will make the following arguments:

(1) Competence: as $q \to +\infty$, which shall be called the *strict domination limit*, the input-to-output mapping $\mathrm{SF}_q(u)$ converges to $\mathrm{SF}_{\mathrm{OT}}(u)$. It has been known since the inception of Optimality Theory that if the constraints have an upper bound, namely, if $q \geq C_k(x) + 1$ for all k and x, then optimisation with respect to $H_q(x)$ yields the same result as optimisation with respect to $H_{\mathrm{OT}}(x)$. We now generalise this observation to unbounded integer-valued constraints: for all $u \in \mathfrak{U}$ there exists a $q_0 \geq 1$ such that $\mathrm{SF}_q(u) = \mathrm{SF}_{\mathrm{OT}}(u)$ for any $q > q_0$. In other words, *q*-HG becomes equivalent to OT for more and more elements of $\mathfrak{U}$, if q is chosen sufficiently large.

(2) Performance: Smolensky and Legendre (2006) suggested viewing the implementation of a grammar with simulated annealing as a model of linguistic performance. This stochastic hill climbing algorithm attempts to find the (global) optimum, but it may be stuck in some other local optima, corresponding to “performance errors”. Therefore, we have run experiments to measure how the probability of producing the grammatical form (the *precision* of the algorithm) depends on parameter q. In the case of a standard HG grammar, if ample time is provided to the algorithm, its probability of producing the grammatical form will converge to 100%. However, we argue that in the strict domination limit the performance of *q*-HG converges towards the performance of OT, and not all of them display the nice behaviour of standard HG.

(3) Speed of production: While larger q values result in more errors in the case of some grammars, the speed of the algorithm improves. Less iterations are required to reach a local optimum, whether it is the global one, or not. It may be advantageous to employ a large q, that is, an OT-like architecture, if fast production is expected, such as on lower linguistic levels. Hence, we argue that phonological and morphological irregularities are a consequence of the brain employing an OT-like grammar for the sake of fast computation.

(4) Language acquisition: We do not present standard learnability results for *q*-HG, but rather suggest that children might change the parameter q as they maturate. An analysis of cluster reduction in Dutch child speech proposes that q is gradually increased in the phonological grammar, which is consistent with the above suggestion that phonology requires large q values for fast computation. At the same time, an analysis of pronoun resolution by children suggests a decreasing q, as the child grows. And indeed, in the syntactic and semantic domain, fewer computations are performed per time unit, and so its speed can be decreased in order to achieve higher precision.

Smolensky and Legendre (2006, vol. 1, p. 87) lists “the emergence of OT’s strict domination constraint interaction (…) from network-level principles” as one of the major open problems in their *Integrated Connectionist/Symbolic Cognitive Architecture*. While it is unclear yet what mechanisms cause the emergence of strict domination in the brain, we now have a hypothesis for what motivates it to happen during maturation.

Reference

Smolensky, Paul and Géraldine Legendre (eds.), *The Harmonic Mind: From Neural Computation to Optimality-Theoretic Grammar*, 2 vols., MIT Press, 2006.